Analytical Model for a WDM Optical Cross-Connect with Limited Conversion Capability

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***Abstract*—In this letter, we introduce a general model for describing the connection setup in various wavelength-routing optical cross-connects with limited conversion capability. It enables us to determine if a given connection pattern can be supported, and to calculate the blocking performance of the cross-connects.**

***Index Terms*—Blocking ratio, connection pattern, limited conversion, optical cross-connect, share-per-node.**

I. INTRODUCTION

WAVELENGTH-ROUTED networks employing wavelength division multiplexing (WDM) have emerged as an attractive solution for wide-area networks [1], [2]. Wavelength conversion can increase the utilization of the link capacity in wavelength-routed networks [3]–[6]. However, wavelength conversion is expensive and this has led to some recent focus on networks with limited wavelength conversion capability [4]–[8].

With limited wavelength conversion in a wavelength-routing optical cross-connect (OXC), a new dimension is added to the conventional blocking problem of circuit switching systems. Consider an OXC with N input and N output links, each carrying W channels at different wavelengths. If full conversion is available, any of the NW input channels can be connected to any of the NW output channels. With limited conversion, not all connection patterns can be established in the OXC. For example, the share-per-node architecture of [4] will only allow up to a certain number of simultaneous wavelength conversions in the OXC. In this paper, we present a novel approach to the modeling of an OXC, in Section II. This model is expected to provide a general framework for the analysis of an OXC with different wavelength conversion constraints. To illustrate this, an analytical model for the performance of a share-per-node OXC is presented in Section III. In Section IV, numerical results for this OXC are presented.

II. GENERAL MODEL FOR CONNECTION SETUP

Connections between the input and output channels in an OXC can be set up subject to the constraint that *each input channel connects to only one output channel*. The overall connection setup can be modeled using an $NW \times NW$ array, where

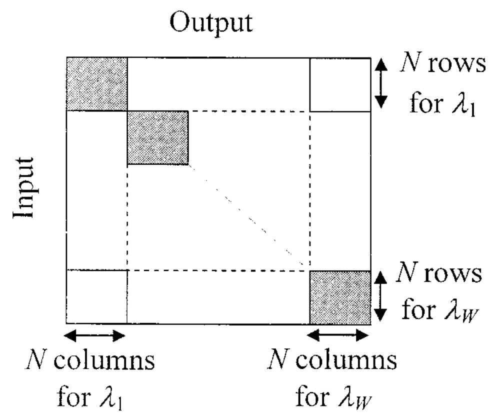

Fig. 1. Array model for connection setup.

row $(k-1) \times N + i$ corresponds to wavelength k of input link i and column $(k-1) \times N + j$ corresponds to wavelength k of output link j. A connection between input channel i and output channel j is represented by placing an object in the position (i, j) of the array subject to the constraints that *a) at most one object is placed in each row and b) at most one object is placed in each column*.

The connections in the OXC are classified based on wavelength continuity. A wavelength-continuous (WC) connection connects an input to an output with the same wavelength, while a nonwavelength-continuous (NWC) connection connects an input (through a wavelength converter) to an output with a different wavelength. For our model, the rows and columns are grouped according to the wavelength, and the positions for WC connections are shown shaded as in Fig. 1.

The constraints mentioned above correspond to those in traditional circuit-switched connections. In a WDM OXC with limited conversion capability, only some of the possible connection patterns can be supported, as determined by its wavelength conversion capability. The array model may then be used to represent various WDM OXC's with different wavelength conversion capabilities by further constraining the object placement mentioned above. Depending on these constraints, a wide variety of OXC's may be represented using this array based approach. Some examples for this are given next.

A. Share-Per-Node Conversion

In a share-per-node WDM OXC [4], V converters are available for use in any connection. Therefore, connection patterns with more than V NWC connections will not be supported. This additional constraint implies that *at most V objects may be*

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placed outside the shaded area if the connection pattern (with all its connections) is supported.

B. Share-Per-Link Conversion

For a share-per-link OXC [4] with V converters per link, we are limited to at most V NWC connections on each output link. The constraint in this case is that *within the W columns for each link, at most V objects may be placed outside the shaded area*.

C. Limited-Range Conversion

In an OXC with limited-range conversion [8], no connection is allowed from a channel with λ_i to a channel with λ_j if λ_i is not convertible to λ_j. The constraint for this is that *no object can be placed in position (r, c) if input r uses λ_i, output c uses λ_j and λ_i is not convertible to λ_j*. The share-per-node OXC architecture provides the greatest flexibility in converter usage. The analysis for this OXC using the array model is presented next in Section III.

III. Analysis of Share-per-Node OXC

Let $y \in \{1, 2, \cdots, NW\}$ be the number of connections to be set up through an OXC. For any y, let $G_0(y)$ and $G(y)$ denote the number of all possible connection patterns and the number of supported connection patterns respectively, where

$$G_0(y) = C_y^{NW} P_y^{NW} \triangleq T_y^{NW} \tag{1}$$

with

$$C_y^n = \frac{n!}{y!(n-y)!} \qquad \text{and} \qquad P_y^n = \frac{n!}{(n-y)!}.$$

Since an unsupported connection pattern cannot use the OXC, we define the Blocking Ratio $B(y)$ as a performance parameter for the OXC for a given value of y, where

$$\text{Blocking Ratio,} \quad B(y) = \frac{G_0(y) - G(y)}{G_0(y)}. \tag{2}$$

Considering the overall performance over all possible y, we define the Overall Blocking Ratio B as

$$\text{Overall Blocking Ratio} \quad B = \frac{\sum_{y=1}^{NW} [G_0(y) - G(y)]}{\sum_{y=1}^{NW} G_0(y)}. \tag{3}$$

To find $G(y)$, the following two cases need to be considered:

Case 1: $1 \le y \le V$

$$G(y) = G_0(y) = T_y^{NW} \quad \text{and} \quad B(y) = 0$$

Case 2: $V + 1 \le y \le NW$.

Let e_j^y be the number of ways to place y objects in the array with exactly j of them in the shaded area. For a connection to be supported, no more than V of the y objects can be placed outside the shaded area, which gives

$$G(y) = \sum_{j=y-V}^{y} e_j^y. \tag{4}$$

A. Counting Objects with Multiple Properties

For the array of Fig. 1, the number of shaded positions is given by $p = N^2 W$. Consider the problem of enumerating $G_0(y)$ objects with the properties $1, 2, \cdots, p$, where a connection pattern is regarded as an object and is said to have the property i if the ith shaded position is occupied [9]. Let $N(i_1, i_2, \cdots, i_k)$ be the number of connection patterns with the properties $i_1, \cdots, i_k$, where $N(i_1, i_2, \cdots, i_k) = T_{y-k}^{NW-k}$.

Define

$$s_k^y \triangleq \sum N(i_1, i_2, \cdots, i_k) = r_k^W T_{y-k}^{NW-k} \tag{5}$$

where $\{i_1, i_2, \cdots, i_k\} \subseteq \{1, 2, \cdots, p\}$ and the sum is taken over all possible combinations of k properties. r_k^W represents the number of ways to place k objects in the shaded area, which consists of W $N \times N$ squares. From the principle of inclusion and exclusion [9], we get

$$e_j^y = \sum_{k=y-V}^{y} (-1)^{k-j} C_{k-j}^k s_k^y. \tag{6}$$

Substituting (6) into (4) and applying (5), we derive that

$$G(y) = \sum_{k=y-V}^{y} (-1)^{k-(y-V)} C_{y-V-1}^{k-1} r_k^W T_{y-k}^{NW-k}. \tag{7}$$

B. Evaluation of r_k^W

Consider a shaded area that comprises m $N \times N$ squares without common row and column. Let $k(i)$ be the number of objects to be placed in the ith square; this can be done in $T_{k(i)}^N$ ways. Then $k(1), k(2), \cdots, k(m)$ objects may be placed in the m squares respectively in $T_{k(1)}^N T_{k(2)}^N \cdots T_{k(m)}^N$ ways with $\sum_{i=1}^m k(i) = k$ and $k(i) \le N$ for $i = 1, 2, \cdots, m$. Using this, we can derive

$$r_k^m = \sum_{i=l}^{L} T_i^N r_{k-i}^{m-1}, \qquad k \le Nm \tag{8}$$

with $l = \max(0, k - N(m-1))$ and $L = \min(k, N)$, and

$$r_0^m = 1, \qquad m = 1, 2, \cdots, W$$

and

$$r_k^1 = T_k^N, \qquad k = 0, 1, \cdots, N$$

With r_k^W calculated from (8), we obtain $G(y)$ for $V + 1 \le y \le NW$, from (7) and as $G(y) = G_0(y) = T_y^{NW}$ for $1 \le y \le V$. Using (1)–(3), we can then calculate the blocking ratio $B(y)$ for a connection pattern with y connections and the overall blocking ratio B over all connection patterns $1 \le y \le NW$.

C. Blocking Probability of a Connection

Let $BC(y)$ represents the total number of blocked connections over all connection patterns when the number of connections in the connection pattern is y. Similarly, let $P_B(y)$ be the probability that a particular connection in connection patterns with y connections is blocked. Then

$$BC(y) = \begin{cases} 0, & \text{for } y \le V \\ \sum_{j=0}^{y-V-1} (y - V - j) e_j^y, & \text{for } y > V \end{cases} \tag{9}$$

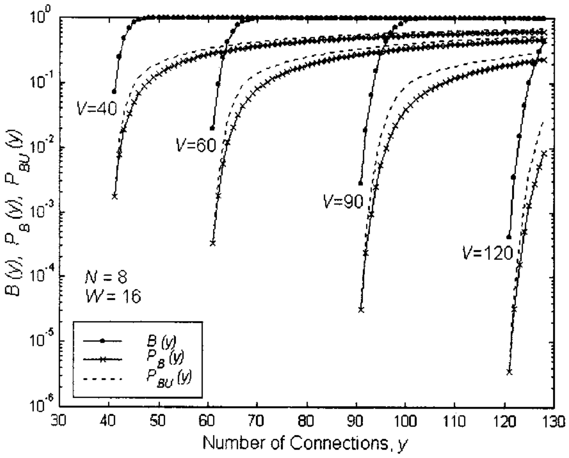

Fig. 2. $B(y)$, $P_B(y)$ and $P_{BU}(y)$ vs. y for $N = 8, W = 16$.

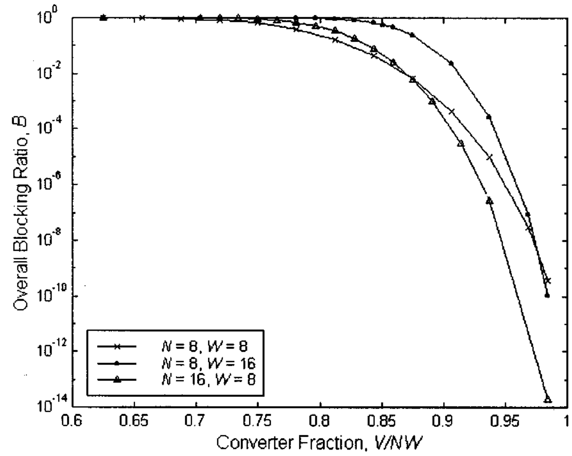

Fig. 3. Overall connection pattern blocking vs. converter fraction.

and $P_B(y) = BC(y)/[yG_0(y)]$. A simple upper bound $P_{BU}(y)$ on $P_B(y)$ may be easily computed as

$$P_{BU}(y) = \begin{cases} 0, & \text{for } y \le V \\ \dfrac{y-V}{y} B(y), & \text{for } y > V \end{cases} \tag{10}$$

We have calculated $P_B(y)$ exactly by simplifying (9) to give

$$P_B(y) = \begin{cases} 0, & \text{for } y \le V \\ \dfrac{1}{yG_0(y)} \left[\sum_{k=0}^{y} (-1)^k s_k^y\right], & \text{for y = V + 1} \\ \dfrac{1}{yG_0(y)} \left[(y-V)s_0^y - s_1^y + \sum_{k=y-V}^{y} (-1)^{k-(y-V)} \right. \\ \quad \left. \cdot \frac{y-V-k}{k-1} C_{y-V-1}^{k-1} s_k^y \right], & \text{for y > V + 1} \end{cases}$$

IV. Numerical Results for Share-per-Node OXC

Fig. 2 shows plots of the blocking ratio $B(y)$ and the probability $P_B(y)$ of a particular connection being blocked (in a connection pattern with y connections) against y for various values of V with $N = 8$ and $W = 16$. For comparison, we have also shown the upper bound $P_{BU}(y)$ in Fig. 2. For any given value of V, both $B(y)$ and $P_B(y)$ start from 0 for $y \le V$, and increase thereafter for increasing y. The overall blocking ratio B, considering all connection patterns for all $1 \le y \le NW$, is shown as a function of the converter fraction $V/(NW)$ in Fig. 3 for different values of N and W. The importance of these performance results lie in the fact that they represent the performance of an individual OXC when it is required to support a certain number of connections. In particular, it can be observed that the blocking ratio of an entire connection pattern or the blocking probability of a particular connection in a connection pattern subject to blocking, may actually be quite poor when the OXC is considered by itself.

Our results show that for a particular OXC in a WDM network, the overall connection pattern for the OXC will become rapidly unsupportable [i.e. high $B(y)$, or B] as the number of connections increases beyond the number of converters V or decreasing $V/(NW)$. This will be true even though an individual connection in the connection pattern may still have a low blocking probability $P_B(y)$; however this also increases with y increasing beyond V. In spite of this trend, the network considered as a whole may not require too many converters at each node because of statistical multiplexing and the fact that many connections through a node may actually not need wavelength conversion. Our results, however do indicate the importance of efficient wavelength assignment in such a network. For the various traffic in such a network, the wavelength assignment and the route selection should be done such that the number of connections requiring wavelength conversion at a node is kept as low as possible. This is in agreement with the observations made in [4] and elsewhere that in a typical WDM network (considered as a whole), the number of converters required per node can be quite low if wavelength assignments are efficiently done.

V. Conclusions

We have proposed here an analytical approach for studying the blocking performances of WDM OXC's. An array based model for representing the connection setup has been proposed, which can flexibly model various kinds of OXC's with limited conversion capabilities by suitably constraining object placement in the array. The share-per-node architecture with limited conversion was analyzed using this approach and performance results for this have been presented.

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